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REPEATED MEASURES: SUGGESTED APPROACH FOR SIMULTANEOUS INDEPENDENT AND DEPENDENT MEASURES

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ABSTRACT

When investigating differences between two sets of scores, the t test is appropriate. If the two sets of data are from two groups of subjects, then the independent t test is appropriate. If the two sets are from the same subjects, the dependent t test is required. In this paper, the authors describe the use of a third test when part of a data set meets independent t assumptions and part meets the dependent t assumptions. Application to a real study is described.

REPEATED MEASURES: SUGGESTED APPROACH FOR SIMULTANEOUS INDEPENDENT AND DEPENDENT MEASURES

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The purpose of this paper is to identify a generalized statistical procedure that can be used in the unique situation when a data set exists that may have some of its subjects repeated (dependent) and some of its subjects mutually exclusive in treatment groups (independent). This paper will present a simple case where this actually occurred and a simultaneously independent and dependent t test of significance was developed. In the final portion of this paper the authors will expand this approach to repeated measures designs with incomplete data.

A widespread use of statistical procedures in behavioral research is testing for significant difference between two or more sets of data. Often the researcher is interested in finding evidence supporting one treatment over others, for example. When examining differences between two sets of scores, the t test is appropriate. To meet the underlying assumptions of the t test, the researcher considers the source of the data. If the two sets of data are from two groups of subjects, then the independent t test is appropriate. Here the amount of sampling variation is independent for each group. If, on the other hand, the

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two sets of data are from the same subjects, the dependent t test would be used. This constitutes a repeated measures design, that is, the measure is repeated on the same group of subjects twice. Therefore, selection of the correct t test is an important one in order to account for subjects' effects when necessary.

In this paper the writers will describe a situation where part of the data meets the dependent t assumptions and part meets the independent t assumptions. In other words, some of the subjects undergo repeated measures and others do not. A technique for accounting for subjects' effects will be described and methodology for application to the research setting will be outlined.

Most traditional statistics textbooks deal with independent and dependent t tests in an either-or fashion, i.e., assuming the data will meet either the independent or dependent assumptions. The situation may not always be clear. For example, some schools may be more insistent upon demands of confidentiality than others. Thus, a researcher may be able to match pre and post test data at one school but not at another; presumably the same individuals are involved at the pre and post testing at the second school, but no matching can take place. This could be so even if the "pre" and "post" tests are administered simultaneously if matching the two sets of tests are not possible at the second school.

Confidentiality concerns not the only likely reason for unmatched pre and post tests. Suppose a long term research project (say three years) is entered at several sites. Any lack of total stability will cause the existence of several subjects measured in the first year who

are unavailable for testing in the third year; similarly, subjects will be available for testing in the third year who were unavailable in the first year. In that the transient population may be a most important aspect of delivering schooling services in a targeted population, their exclusion for data gathering reasons is indefensible. The quandry becomes: are independent t test or dependent t test procedures called for in comparing the effects of an intervention? To attempt an answer, consider the following example.

Subjects 1 through 7 are in both treatments. Others are in treatment one only (subjects 18 through 25). And others are in treatment two only (subjects 10 through 17). Data from subjects 1 through 7 meet, then, the assumptions of the dependent t test, a repeated measures situation. Comparing treatment one data to treatment two data from subjects 10 through 17, and 15 through 25, however, is comparing two independent groups, i.e., an independent t test. (See Table 1).

A third solution is based in the general linear models procedures. Writing regression equations facilitates an effective solution here. The models are:

FULL
MODEL: $Y = a_o u + a_1 T_1 + a_2 P_1 . . . a_2 P_7 + E$

RESTRICTED
MODEL: $Y = a_o u + a_1 P_1 . . . a_7 P_7 + E$

N = 25 (32 replicates)

linearly independent vectors (full model) = 8
linearly independent vectors (restricted model) = 7

$df_n = 1$ $df_d = 22$

Table 1

Data for a Mixed Two Treatment Situation

Subject	Y	U	T_1	T_2	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1	13	1	1	0	1	0	0	0	0	0	0
1	17	1	0	1	1	0	0	0	0	0	0
2	14	1	1	0	0	1	0	0	0	0	0
2	18	1	0	1	0	1	0	0	0	0	0
3	15	1	1	0	0	0	1	0	0	0	0
3	19	1	0	1	0	0	1	0	0	0	0
4	12	1	1	0	0	0	0	1	0	0	0
4	14	1	0	1	0	0	0	1	0	0	0
5	17	1	1	0	0	0	0	0	1	0	0
5	17	1	0	1	0	0	0	0	1	0	0
6	19	1	1	0	0	0	0	0	0	1	0
6	20	1	0	1	0	0	0	0	0	1	0
7	20	1	1	0	0	0	0	0	0	0	1
7	23	1	0	1	0	0	0	0	0	0	1
8	13	1	1	0	0	0	0	0	0	0	0
9	13	1	1	0	0	0	0	0	0	0	0
10	13	1	1	0	0	0	0	0	0	0	0
11	18	1	1	0	0	0	0	0	0	0	0
12	24	1	1	0	0	0	0	0	0	0	0
13	15	1	1	0	0	0	0	0	0	0	0
14	11	1	1	0	0	0	0	0	0	0	0
15	12	1	1	0	0	0	0	0	0	0	0
16	17	1	0	1	0	0	0	0	0	0	0
17	13	1	0	1	0	0	0	0	0	0	0
18	17	1	0	1	0	0	0	0	0	0	0
19	21	1	0	1	0	0	0	0	0	0	0
20	24	1	0	1	0	0	0	0	0	0	0
21	17	1	0	1	0	0	0	0	0	0	0
22	18	1	0	1	0	0	0	0	0	0	0
23	19	1	0	1	0	0	0	0	0	0	0
24	23	1	0	1	0	0	0	0	0	0	0
25	19	1	0	1	0	0	0	0	0	0	0

Had two separate analysis been performed on the data, one for the first seven subjects (a related t test) and the second for the last 18 subjects, then the t values would have respectively been $t = 4.204$ ($p < .01$) and $t = 2.23$ ($p < .05$), both tests showing significance in favor of treatment one. Testing for the significance of this restriction is accomplished by:

$$F = \frac{(R_F^2 - R_R^2)/1}{(1 - R_F^2)/df_2}, \text{ where } R_F^2$$

refers to the multiple correlation squared from the full model, R_R^2 refers to the multiple correlation squared from the restricted model and df_w refers to the degrees of freedom from the deviation from regression (error term) for the full model:

$$F = \frac{(.44111 - .22899)/1}{(1 - .44111)/23} = 8.730,$$

since there is one degree of freedom in the numerator, $t = \sqrt{F} = 2.955$ ($p < .01$), again in favor of the first treatment.

One can see from the preceding regression models that the variability due to the individual differs from subjects that are in both treatments and those in one treatment. The variability of the former is controlled for by person vectors and conceptually eliminated from the error term. The subjects we do not have repeated measures on (subjects 8 through 25) and therefore cannot estimate within subject variability are not subtracted from the error term. The degrees of freedom are correct for the independent and dependent t-test situations.

If one ran this once for an independent t-test situation and then again for a dependent t-test situation, the N would vary and the

power would be reduced for the second run. Consider the following power analyses, and compare with the general linear model method.

Independent t test

N = 18

df = 16

Power analysis for α = .05 and large effect size (.35) $\simeq$ .61

Dependent t test

N = 7

df = 6

Power analysis for α = .05 and large effect size (.35) $\simeq$ .30

General Linear Model Method

N = 25 (32 replicates)

df = 22

Power analysis for α = .05 and large effect size (.35) $\simeq$ .78

It is readily apparent that the general linear model approach is more powerful than either of the other two methodologies.

Because in the applied research setting the preceding dilemma may occur, we suggest that this method may be a tool that is of potential use for the practitioner. It supports the notion that most of the statistical techniques (X^2, t, F, etc.--Newman, 1976; Kerlinger & Pedhazur, 1973; McNeil, Kelly, & McNeil, 1975; Cohen & Cohen, 1975; Draper & Smith, 1966, Newman & Benz, 1979) are calculated based upon

the general least squares solution. If taught in this manner rather than with the traditional analysis of variance approach, understanding and practical value are improved.

Repeated Measures Design
with Incomplete Data

When completing an experiment where each subject is to be measured several times, it is not always possible, or even advisable, to measure each subject on each treatment. For example, Benz (1979) used six treatment conditions in measuring role expectations of public school teachers. Because of the necessity of six treatment conditions along with the desirability of limiting respondent's time in undergoing treatments, six treatments were randomly assigned to the subjects with three treatments assigned to each subject. Note that if a design is anticipated where all possible combinations of three treatment conditions are included, then a minimum of $6 \cdot 5 \cdot 4 = 120$ subjects would be needed just to have a single subject in each treatment combination. Clearly, the number of subjects available to a given researcher would seldom be a convenient multiple of 120. Further, if it is necessary to include several subjects in at least part of the same treatment combinations, then an alternative analysis would seem to be useful. Also, if fewer than 120 subjects are available, an alternative analysis is necessary. Benz (1979) had only 25 subjects available and thus needed an alternative analysis. She assigned treatment conditions to small groups of teachers; a sample representation of a proto-type of Benz's analysis is given in Table 2.

Table 2

Scores for 25 Subjects in Three Treatment Combinations

Subject	Treatment One	Two	Three	Four	Five	Six
1	12	13		17		
2	13	14		18		
3	14	15		19		
4		12		14		16
5		17		17		17
6		19		20		21
7		20		23		25
8	10			13	16	
9	12			17	19	
10			20	21	25	
11				24	23	25
12				17	25	33
13	15		18	18		
14	17		18	19		
15			18	23	37	
16			15	19	29	
17			14	17		35
18	12	13	11			
19	13	13	22			
20	13	13	25			
21	15	18	33			
22		24			24	24
23		15			17	34
24		11			14	17
25		12			14	31

To form linear models for the data in Table 2, the following variables can be defined:

Y = the criterion score

X_1 = 1 if the treatment condition is treatment 1, 0 otherwise;

X_2 = 1 if the treatment condition is treatment 2, 0 otherwise;

X_3 = 1 if the treatment condition is treatment 3, 0 otherwise;

X_4 = 1 if the treatment condition is treatment 4, 0 otherwise;

X_5 = 1 if the treatment condition is treatment 5, 0 otherwise;

X_6 = 1 if the treatment condition is treatment 6, 0 otherwise;

S_1 = 1 if the score is from subject 1, 0 otherwise; and

S_2 to S_{25} are subject binary variables similar to S_1 for subject 2 to subject 25.

Table 3 gives the design matrix for the first five subjects; subjects 6 through 25 can similarly be shown in a design matrix.

The use of three linear models is useful to analyze the data given in Table 2:

Full Model: $Y = b_o+b_1X_1+b_2X_2+b_3X_3+b_4X_4+b_5X_5+b_6S_1+S_2+\ldots b_{29}S_{24}+e_1.$ (1)

Restricted Model: $Y = bo+b_6S_1+b_7S_2+\ldots+b_{29}S_{24}+e_2.$ (2)

Treatment Model: $Y = bo+b_1X_1+b_2X_2+b_3X_3+b_4X_4+b_5X_5+e_3.$ (3)

In equations 1 through 3, the b_1s are regression coefficients; also, the b_1s with the same subscript are likely to vary from equation to equation. Table 4 contains a summary table for the use of equations 1 to 3 with the data in Table 2.

Table 3

Design Matrix for the First Five Subjects from Table 2

Subject	Y	X_1	X_2	X_3	X_4	X_5	X_6	S_1	S_2	S_3	S_4	S_5	S_6	...	S_{25}
1	12	1	0	0	0	0	0	1	0	0	0	0	0		0
1	13	0	1	0	0	0	0	1	0	0	0	0	0		0
1	17	0	0	0	1	0	0	1	0	0	0	0	0		0
2	13	1	0	0	0	0	0	0	1	0	0	0	0		0
2	14	0	1	0	0	0	0	0	1	0	0	0	0		0
2	18	0	0	0	1	0	0	0	1	0	0	0	0		0
3	14	1	0	0	0	0	0	0	0	1	0	0	0		0
3	15	0	1	0	0	0	0	0	0	1	0	0	0		0
3	19	0	0	0	1	0	0	0	0	1	0	0	0		0
4	12	0	1	0	0	0	0	0	0	0	1	0	0		0
4	14	0	0	0	1	0	0	0	0	0	1	0	0		0
4	16	0	0	0	0	0	1	0	0	0	1	0	0		0
5	17	0	1	0	0	0	0	0	0	0	0	1	0		0
5	17	0	0	0	1	0	0	0	0	0	0	1	0		0
5	17	0	0	0	0	0	1	0	0	0	0	1	0		0

Table 4

Summary Table for a Repeated Measure Design with Incomplete Data

Source of Variation	df	SS	MS	F	R^2
Full Model	29	1898.98			.67335
Subjects	24	1211.51			.42959
Treatments	5	1107.45			.39269
Treatments (Independent of Subjects)	5	687.47	137.45	6.71	.24376
Error	45	921.20	20.47		
Total	74	2820.18			

The solution is additive in the same sense that a hierarchical model (Cohen, 1968) is additive. Measuring the treatment effect without reference to the subjects yields a sum of squares different from the treatment effect independent of subjects.

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